Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number | Candidate Number

Time 2 hours

Paper reference **8MA0/01**

Mathematics

Advanced Subsidiary
PAPER 1: Pure Mathematics

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 16 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over

1. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Using algebra, solve the inequality

$$x^2 - x > 20$$

writing your answer in set notation.

(3)

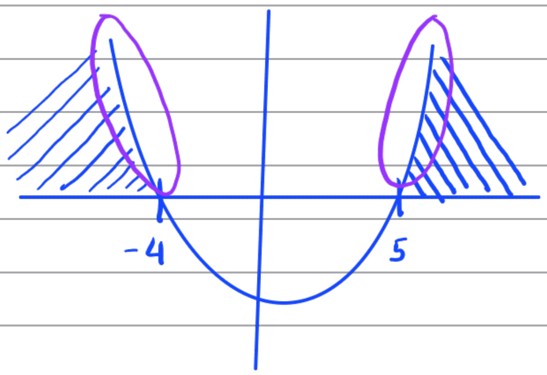
$x^2 - x > 20$

$= x^2 - x - 20 > 0$ — Put the value into calculator

$= (x-5)(x+4) > 0$

$x > 5$ and $x < -4$ ①

Critical values $x = 5$ and $x = -4$



①

inequality satisfied when $x < -4$ and $x > 5$

In set notation :

$\{x : x < -4\} \cup \{x : x > 5\}$ ①

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Question 1 continued

(Total for Question 1 is 3 marks)

2. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

Given

$$\frac{9^{x-1}}{3^{y+2}} = 81$$

express y in terms of x, writing your answer in simplest form.

(3)

$\frac{9^{x-1}}{3^{y+2}} = 81 \longrightarrow \frac{3^{2x-2}}{3^{y+2}} = 3^4$ 1

$= 3^{2x-2-(y+2)} = 3^4$

We compare powers :

$= 2x-2-y-2 = 4$ 1

$y = 2x-8$ 1

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Question 2 continued

(Total for Question 2 is 3 marks)

3. Find

$$\int \frac{3x^4 - 4}{2x^3} \, dx$$

writing your answer in simplest form.

(4)

$$\int \frac{3x^4 - 4}{2x^3} \, dx = \int \frac{3x^4}{2x^3} - \frac{4}{2x^3} \, dx \quad (1)$$

$$= \int \frac{3}{2}x - 2x^{-3} \, dx \quad (1)$$

$$= \frac{3}{2} \times \frac{x^2}{2} - 2 \times \frac{x^{-2}}{2} + c \quad (1)$$

$$= \frac{3}{4}x^2 + \frac{1}{x^2} + c \quad (1)$$

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Question 3 continued

(Total for Question 3 is 4 marks)

4. [*In this question the unit vectors* **i** *and* **j** *are due east and due north respectively*.]

A stone slides horizontally across ice.

Initially the stone is at the point $A(-24\mathbf{i} - 10\mathbf{j})$ m relative to a fixed point O.

After 4 seconds the stone is at the point $B(12\mathbf{i} + 5\mathbf{j})$ m relative to the fixed point O.

The motion of the stone is modelled as that of a particle moving in a straight line at constant speed.

Using the model,

(a) prove that the stone passes through O,

(2)

(b) calculate the speed of the stone.

(3)

a) The position vectors are scalar multiples of each other

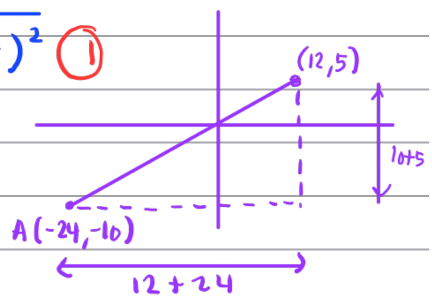
$(-24\underline{i} - 10\underline{j}) = -2(12\underline{i} + 5\underline{j})$ 1

$\overrightarrow{AO} = -2(\overrightarrow{OB})$

Hence, the vectors $\overrightarrow{AO}$ and $\overrightarrow{OB}$ are parallel, and as the stone is travelling in a straight line $\overrightarrow{AB}$, the stone passes through the point O as $\overrightarrow{AB}$ does. 1

b) The distance AB $= \sqrt{(12+24)^2 + (10+5)^2}$ 1

$= 39$ m



The speed of the stone $= \dfrac{39}{4}$ 1 $= 9.75$ m/s 1

Question 4 continued

(Total for Question 4 is 5 marks)

5.

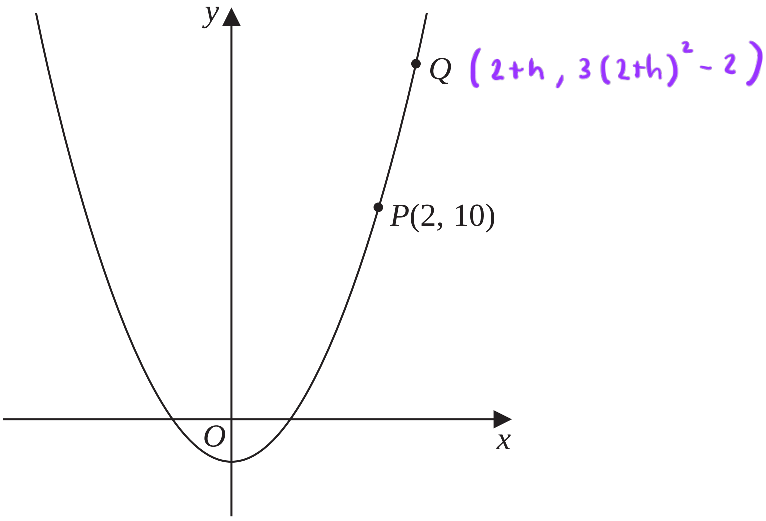


Figure 1

Figure 1 shows part of the curve with equation $y = 3x^2 - 2$

The point $P(2, 10)$ lies on the curve.

(a) Find the gradient of the tangent to the curve at P.

(2)

The point Q with x coordinate $2 + h$ also lies on the curve.

(b) Find the gradient of the line PQ, giving your answer in terms of h in simplest form.

(3)

(c) Explain briefly the relationship between part (b) and the answer to part (a).

(1)

a) $y = 3x^2 - 2$

$\Rightarrow \frac{dy}{dx} = 6x$ At P : $\frac{dy}{dx} = 6(2) = 12$ 1

Gradient of tangent at P is 12. 1

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Question 5 continued

b) Coordinates at Q = $(2+h,\ 3(2+h)^2-2)$

$$\frac{\Delta y}{\Delta x} = \text{Gradient } PQ = \frac{3(2+h)^2-2-10}{(2+h)-2} \quad (1)$$

$$= \frac{3(2+h)^2-12}{(2+h)-2} = \frac{12h+3h^2}{h} \quad (1)$$

$$= 12+3h \quad (1)$$

c) As $h \to 0$, the gradient PQ, $12+3h \to 12$.

So, as Q gets closer to P, the gradient of the chord tends toward the instantaneous gradient of the curve at P. (1)

(Total for Question 5 is 6 marks)

6. **In this question you should show all stages of your working.**

Solutions relying on calculator technology are not acceptable.

(a) Using algebra, find all solutions of the equation

$$3x^3 - 17x^2 - 6x = 0$$

(3)

(b) Hence find all real solutions of

$$3(y-2)^6 - 17(y-2)^4 - 6(y-2)^2 = 0$$

(3)

a)

$3x^3 - 17x^2 - 6x = 0$

$\Rightarrow x(3x^2 - 17x - 6) = 0$ 1

$\Rightarrow x(3x+1)(x-6) = 0$ 1

So, the solutions are $x = 0, -\frac{1}{3}, 6$ 1

b)

Let $n = (y-2)^2$, then

$3(y-2)^6 - 17(y-2)^4 - 6(y-2)^2 = 0$ 1

$3n^3 - 17n^2 - 6n = 0$ has the solutions :

$n = 0, -\frac{1}{3}, 6$ from part (a)

except $n \neq -\frac{1}{3}$ as $n \geq 0$ (as it is squared)

$\Rightarrow (y-2)^2 = 0$ and $(y-2)^2 = 6$

which gives the solutions

$y = 2$ 1 and $y = 2 \pm \sqrt{6}$ 1

Question 6 continued

(Total for Question 6 is 6 marks)

7. A parallelogram $PQRS$ has area $50\,\text{cm}^2$

Given

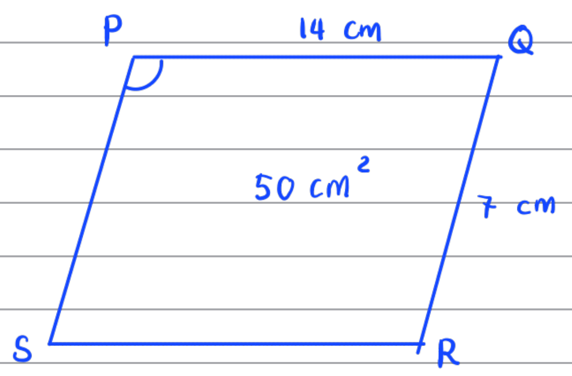
- PQ has length 14 cm
- QR has length 7 cm
- angle SPQ is obtuse

find

(a) the size of angle SPQ, in degrees, to 2 decimal places,
(3)

(b) the length of the diagonal SQ, in cm, to one decimal place.
(2)

a)



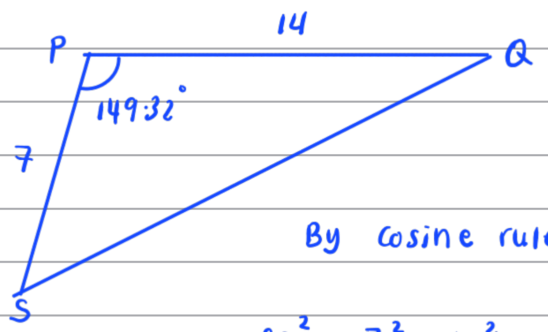
$50 = 7 \times 14 \sin(\angle SPQ)$ 1

$\Rightarrow \angle SPQ = 180 - \arcsin\left(\frac{50}{98}\right)$ 1

$\angle SPQ = 149.32°$ 1

$\angle SPQ$ is obtuse. Since $\arcsin\left(\frac{50}{98}\right)$ gives us acute angle, we substract 180° with the solution of $\arcsin\left(\frac{50}{98}\right)$.

b)



By cosine rule :

$SQ^2 = 7^2 + 14^2 - 2(7)(14)\cos(149.32°)$ 1

$SQ^2 = 413.57046$

$SQ = 20.3$ cm 1

Question 7 continued

(Total for Question 7 is 5 marks)

8. $g(x) = (2 + ax)^8$ where a is a constant

Given that one of the terms in the binomial expansion of $g(x)$ is $3402x^5$

(a) find the value of a.

(4)

Using this value of a,

(b) find the constant term in the expansion of

$$\left(1 + \frac{1}{x^4}\right)(2 + ax)^8$$

(3)

a)

By the binomial expansion, the x^5 term is :

BINOMIAL SERIES :

$(a+b)^n = a^n + \binom{n}{1} a^{n-1} \times b + \ldots\ldots + b^n$

$$\binom{8}{5} \times 2^{8-5} (ax)^5$$

$$= \frac{8!}{5!(8-5)!} \times 2^3 \times a^5 x^5 \quad 1$$

where : $\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$

$$= 448a^5x^5 = 3402x^5 \quad 1$$

$$= 448a^5 = 3402$$

$$: a^5 = \frac{3402}{448} = \frac{243}{32} \quad 1$$

$$a = \frac{\sqrt[5]{243}}{\sqrt[5]{32}}$$

$$a = \frac{3}{2} \quad 1$$

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Question 8 continued

b)

The first constant is 2^8 1 (the constant of the expansion $\times 1$) = 256.

The second constant is the x^4 term of the expansion $\times \frac{1}{x^4}$ term.

$= {}^8C_4 \times 2^4 a^4 = 70 \times 16 \times \frac{181}{16} = 5670$

1

The constant term is $256 + 5670 = 5926$ 1

(Total for Question 8 is 7 marks)

9. Find the value of the constant k, $0 < k < 9$, such that

$$\int_k^9 \frac{6}{\sqrt{x}}\,dx = 20$$

(4)

$$\int_k^9 \frac{6}{\sqrt{x}}\,dx = \int_k^9 6x^{-1/2}\,dx = 20$$

$$= \left[12x^{1/2}\right]_k^9 = 20 \quad (1)$$

$$= 12 \times \sqrt{9} - 12\sqrt{k} = 20$$

$$= 12 \times 3 - 12\sqrt{k} = 20$$

$$12\sqrt{k} = 36 - 20 \quad (1)$$

$$\sqrt{k} = \frac{16}{12} = \frac{4}{3} \quad (1)$$

$$k = \frac{16}{9} \quad (\text{as } 0 < k < 9) \quad (1)$$

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Question 9 continued

(Total for Question 9 is 4 marks)

10. A student is investigating the following statement about natural numbers.

"$n^3 - n$ is a multiple of 4"

(a) Prove, using algebra, that the statement is true for all odd numbers.
(4)

(b) Use a counterexample to show that the statement is not always true.
(1)

a)

If n is odd, then $n = 2k + 1$ 1 for an integer k.

$n^3 - n = (2k+1)^3 - (2k+1)$ 1

$= 8k^3 + 12k^2 + 6k + 1 - (2k+1)$ 1

$= 8k^3 + 12k^2 + 4k = 4(2k^3 + 3k^2 + k)$

As k is an integer, $4 \times (2k^3 + 3k^2 + k)$ is a multiple of 4.
Therefore, if n is odd then $n^3 - n$ is a multiple of 4. 1

b)

Let $n = 2$.

$2^3 - 2 = 8 - 2 = 6$ which is not a multiple of 4. 1

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Question 10 continued

Question 10 continued

Question 10 continued

(Total for Question 10 is 5 marks)

11. The owners of a nature reserve decided to increase the area of the reserve covered by trees.

Tree planting started on 1st January 2005.

The area of the nature reserve covered by trees, A km^2, is modelled by the equation

$$A = 80 - 45e^{ct}$$

where c is a constant and t is the number of years after 1st January 2005.

Using the model,

(a) find the area of the nature reserve that was covered by trees just before tree planting started.

(1)

On 1st January 2019 an area of 60 km^2 of the nature reserve was covered by trees.

(b) Use this information to find a complete equation for the model, giving your value of c to 3 significant figures.

(4)

On 1st January 2020, the owners of the nature reserve announced a long-term plan to have 100 km^2 of the nature reserve covered by trees.

(c) State a reason why the model is not appropriate for this plan.

(1)

Before tree is planted, $t = 0$

a)

When $t = 0$, $e^{ct} = 1$

$A = 80 - 45 \times 1 = 35$ km^2 (1)

$2005 \longrightarrow 2019 = 14$ years

b) $t = 14$ years. So,

$A = 80 - 45e^{14c} = 60$ (1)

$\Rightarrow \quad 45e^{14c} = 20$ (1)

$\Rightarrow \quad c = \frac{1}{14} \ln\left(\frac{20}{45}\right)$

$c = -0.0579235....$ (1)

$A = 80 - 45e^{-0.0579t}$ (1) (c to 3 sig. fig.)

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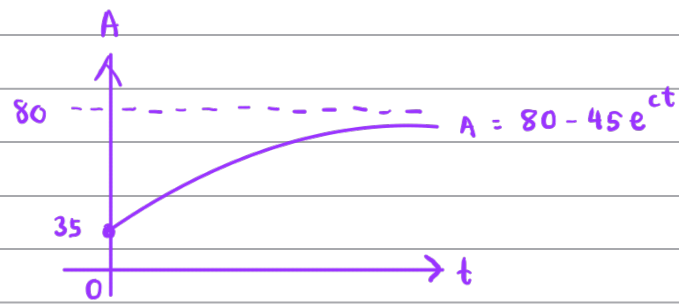
Question 11 continued



c) The maximum area found by the model is 80 km^2, as $t \to \infty$. 1

(Total for Question 11 is 6 marks)

12. **In this question you should show all stages of your working.**

Solutions relying entirely on calculator technology are not acceptable.

(i) Solve, for $0 < \theta \leqslant 450°$, the equation

$$5\cos^2\theta = 6\sin\theta$$

giving your answers to one decimal place.

(5)

(ii) (a) A student's attempt to solve the question

"Solve, for $-90° < x < 90°$, the equation $3\tan x - 5\sin x = 0$"

is set out below.

$$3\tan x - 5\sin x = 0$$
$$3\frac{\sin x}{\cos x} - 5\sin x = 0$$
$$3\sin x - 5\sin x\cos x = 0$$
$$3 - 5\cos x = 0$$
$$\cos x = \frac{3}{5}$$
$$x = 53.1°$$

Identify two errors or omissions made by this student, giving a brief explanation of each.

(2)

The first four positive solutions, in order of size, of the equation

$$\cos(5\alpha + 40°) = \frac{3}{5}$$

are α_1, α_2, α_3 and α_4

(b) Find, to the nearest degree, the value of α_4

(2)

(i) As $\cos^2\theta = 1 - \sin^2\theta$ 1

Then, $5\cos^2\theta = 6\sin\theta$

$5(1-\sin^2\theta) = 6\sin\theta$

$5 - 5\sin^2\theta = 6\sin\theta$

$5\sin^2\theta + 6\sin\theta - 5 = 0$ 1

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Question 12 continued

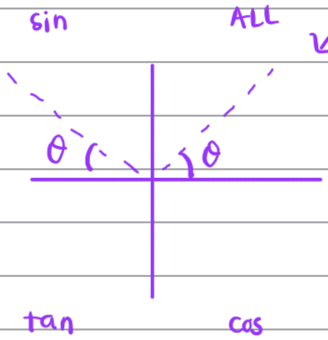
We can use quadratic formula for $ax^2 + bx + c = 0$,
where $a = 5$, $b = 6$, $c = -5$ and $x = \sin\theta$

This gives us

$$\sin\theta = \frac{-6 \pm \sqrt{6^2 - 4(5)(-5)}}{2(5)}$$

we only take the positive root of the quadratic in $\sin\theta$ because negative root gives no solution (1) as $\sin\theta \neq -ve$ values.

$$\sin\theta = \frac{-6 \pm 2\sqrt{34}}{2(5)} = \frac{-3 \pm \sqrt{34}}{5}$$



sin θ can only be positive

$\theta = 34.484\ldots, (180° - 34.484°), (360° + 34.484°)$

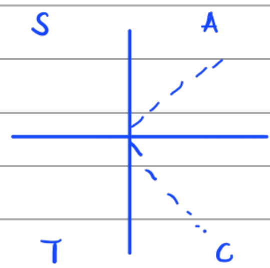
$\theta = 34.484°\ldots, 145.516°\ldots, 394.484°\ldots$

$\theta = 34.5°, 145.5°, 394.5°$ (1 d.p.) (2)

(ii) (a) They cancel out $\sin x$ on line 4 and so miss the solution
$\sin x = 0 \rightarrow x = 0$ (1)

They do not find all the solutions to $\cos x = \frac{3}{5}$ in the given range, they miss the solution $x = -53.1°$. (1)

CAST diagram



(ii) (b) By a CAST diagram, we know these are the solutions for $\cos(5\alpha + 40) = 3/5$

at $5\alpha_1 + 40° = 53.1°$
$5\alpha_2 + 40° = 360° - 53.1°$
$5\alpha_3 + 40° = 360° + 53.1°$
$\Rightarrow 5\alpha_4 + 40° = 720° - 53.1°$ (1)
$5\alpha_4 = 720° - 53.1° - 40° = 626.9°$
$\alpha_4 = 125°$ (nearest degree) (1)

Question 12 continued

Question 12 continued

(Total for Question 12 is 9 marks)

13.

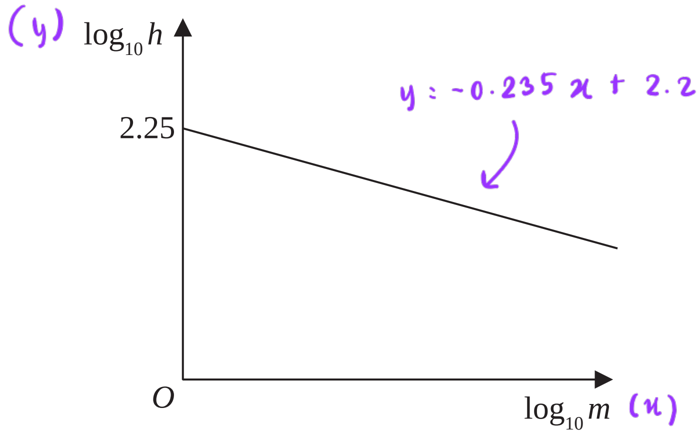


Figure 2

The resting heart rate, h, of a mammal, measured in beats per minute, is modelled by the equation

$$h = pm^q$$

where p and q are constants and m is the mass of the mammal measured in kg.

Figure 2 illustrates the linear relationship between $\log_{10} h$ and $\log_{10} m$

The line meets the vertical $\log_{10} h$ axis at 2.25 and has a gradient of –0.235

(a) Find, to 3 significant figures, the value of p and the value of q.

(3)

A particular mammal has a mass of 5 kg and a resting heart rate of 119 beats per minute.

(b) Comment on the suitability of the model for this mammal.

(3)

(c) With reference to the model, interpret the value of the constant p.

(1)

a) As the relationship between $\log_{10} h$ and $\log_{10} m$ is linear, we can write it as $y = mx + c$

where $x = \log_{10} m$, $y = \log_{10} h$, $m = -0.235$, $c = 2.25$

$\Rightarrow \log_{10} h = -0.235 \log_{10} m + 2.25$

$h = 10^{-0.235 \log_{10} m} \times 10^{2.25}$

$h = m^{-0.235} \times 10^{2.25}$ ① $= pm^q$

$p = 10^{2.25}$ ① $= 178$ and $q = -0.235$ ①

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Question 13 continued

b) If $m = 5$ kg, then the model predicts

$h = 178 \times 5^{-0.235}$ (1) $= 122$ (1) beats per minute

This is accurate to the measured heart (1) rate within 2 significant figures. So, the model is suitable.

c) p would be the resting heart rate in bpm of a mammal with a mass of 1 kg. (1)

Question 13 continued

Question 13 continued

(Total for Question 13 is 7 marks)

14. A curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = -3x^2 + 12x + 8$$

(a) Write $\mathrm{f}(x)$ in the form

$$a(x + b)^2 + c$$

where a, b and c are constants to be found.

(3)

The curve C has a maximum turning point at M.

(b) Find the coordinates of M.

(2)

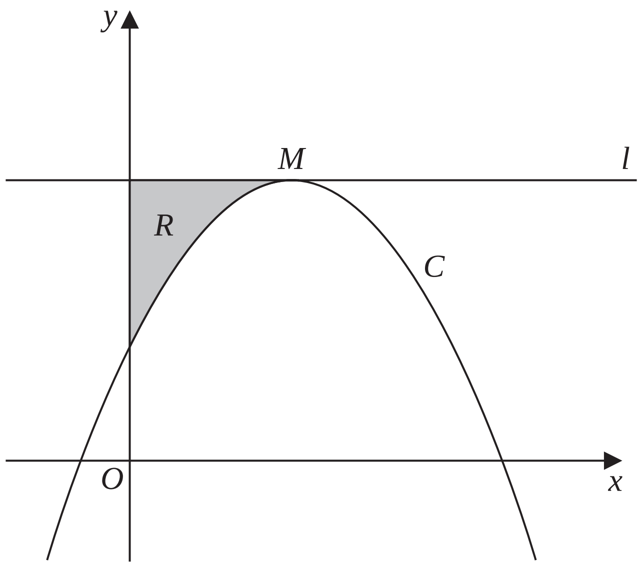

Figure 3

Figure 3 shows a sketch of the curve C.

The line l passes through M and is parallel to the x-axis.

The region R, shown shaded in Figure 3, is bounded by C, l and the y-axis.

(c) Using algebraic integration, find the area of R.

(5)

a) $\mathrm{f}(x) = -3x^2 + 12x + 8$

$= -3(x^2 - 4x) + 8$ (1)

$= -3\left[(x-2)^2 - 4\right] + 8$ (1)

$= -3(x-2)^2 + 20$ (1)

b) By using formula from (a),

when $x = 2$, $y = 20$.

so, $M = (2, 20)$ (1)

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Question 14 continued

c) The line l is $y = 20$. The area under the line l between the y-axis and M is $20 \times 2 = 40$.

So, the area of R is 40 − area under the curve C between $x = 0$ and $x = 2$.

$$R = 40 - \int_0^2 -3x^2 + 12x + 8 \, dx \quad (1)$$

$$R = 40 - (1)\left[-x^3 + 6x^2 + 8x\right]_0^2 \quad (1)$$

$$R = 40 - \left(-2^3 + 6(2)^2 + 8(2)\right) \quad (1)$$

$$R = 40 - 32 = 8 \quad (1)$$

Question 14 continued

Question 14 continued

(Total for Question 14 is 10 marks)

15.

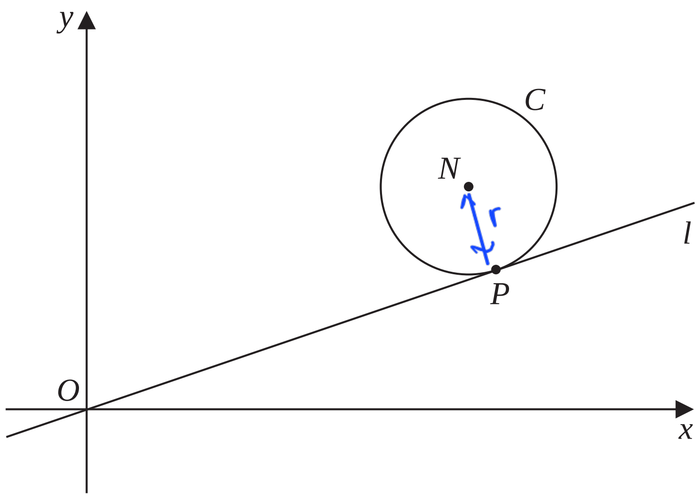

Figure 4

Figure 4 shows a sketch of a circle C with centre $N(7, 4)$

The line l with equation $y = \frac{1}{3}x$ is a tangent to C at the point P.

Find

$m_l = \frac{1}{3}$

(a) the equation of line PN in the form $y = mx + c$, where m and c are constants,

(2)

(b) an equation for C.

(4)

The line with equation $y = \frac{1}{3}x + k$, where k is a non-zero constant, is also a tangent to C.

(c) Find the value of k.

$m_{PN} = \frac{-1}{m_l}$

(3)

a) PN is perpendicular to l, so $m = -3$, and as N is on the line we have point $(7, 4)$

$\Rightarrow y - 4 = -3(x - 7)$ 1

$y = -3x + 25$ 1

b) The radius of C, r = length NP

P is the intersect of $y = \frac{1}{3}x$ and $y = -3x + 25$

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Question 15 continued

At P : $\frac{1}{3}x = -3x + 25$ 1

$x = -9x + 75$

$10x = 75 \Rightarrow x = 7.5$

$y = \frac{1}{3}(7.5) = 2.5 \quad \therefore P(7.5, 2.5)$ 1

Length PN $= \sqrt{(7.5-7)^2 + (4-2.5)^2} = \sqrt{5/2}$ 1

C : $(x-7)^2 + (y-4)^2 = 5/2$ 1

c) When $y = \frac{1}{3}x + k$ satisfies the equation for C

$(x-7)^2 + \left(\frac{1}{3}x + k - 4\right)^2 = \frac{5}{2}$

$x^2 - 14x + 49 + \frac{1}{9}x^2 + \frac{k}{3}x - \frac{4}{3}x + \frac{k}{3}x + k^2 - 4k$
$- \frac{4}{3}x - 4k + 16 = \frac{5}{2}$

$\Rightarrow \frac{10}{9}x^2 + \left(\frac{2}{3}k - \frac{50}{3}\right)x + k^2 - 8k + \frac{125}{2} = 0$ 1

This quadratic must only have one solution, as the tangent only meets the circle once.

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow \left(\frac{2}{3}k - \frac{50}{3}\right)^2 - 4\left(\frac{10}{9}\right) \times \left(k^2 - 8k + \frac{125}{2}\right) = 0$ 1

$= \frac{4}{9}k^2 - \frac{200}{9}k + \frac{2500}{9} - \frac{40}{9}k^2 + \frac{320}{9}k - \frac{2500}{9} = 0$

$= 4k^2 - 200k - 40k^2 + 320k = 0$

Question 15 continued

$\Rightarrow -36k^2 + 120k = 0$

$k = 0$ is the case for line L.

$-36k + 120 = 0$

$k = \frac{120}{36} = \frac{10}{3}$ for the non-zero constant 1

equation: $y = \frac{1}{3}x + \frac{10}{3}$

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Question 15 continued

(Total for Question 15 is 9 marks)

16. The curve C has equation $y = \mathrm{f}(x)$ where

$$\mathrm{f}(x) = ax^3 + 15x^2 - 39x + b$$

and a and b are constants.

Given

- the point (2, 10) lies on C
- the gradient of the curve at (2, 10) is –3

(a) (i) show that the value of a is –2

(ii) find the value of b.

(4)

(b) Hence show that C has no stationary points.

(3)

(c) Write $\mathrm{f}(x)$ in the form $(x - 4)\mathrm{Q}(x)$ where $\mathrm{Q}(x)$ is a quadratic expression to be found.

(2)

(d) Hence deduce the coordinates of the points of intersection of the curve with equation

$$y = \mathrm{f}(0.2x)$$

and the coordinates axes.

(2)

(a)

(i) $\frac{dy}{dx} = 3ax^2 + 30x - 39$

$\frac{dy}{dx} = -3$ when $x = 2$ $\Rightarrow$ $-3 = 3a(2)^2 + 30(2) - 39$ (1)

$12a = -24$

$a = -2$ (1)

(ii) As $f(2) = 10$

$\Rightarrow (-2)(2)^3 + 15(2)^2 - 39(2) + b = 10$ (1)

$-16 + 60 - 78 + b = 10$

$b = 44$ (1)

(b) $f'(x) = -6x^2 + 30x - 39$ (1)

$b^2 - 4ac \Rightarrow 30^2 - 4(-6)(-39) = -36 < 0$

(1)

Since $b^2 - 4ac < 0$, $f'(x) \neq 0$ so no stationary point exists. (1)

P 6 6 5 8 5 A 0 4 2 4 4

Question 16 continued

c) $f(x) = -2x^3 + 15x^2 - 39x + 44$

$f(x) = (x-4)\,Q(x)$

$$
\begin{array}{r l}
 & -2x^2 + 7x - 11 \quad 1 \\
x-4 \,) & -2x^3 + 15x^2 - 39x + 44 \\
- & -2x^3 + 8x^2 \\
 & \qquad 7x^2 - 39x \\
 & - \; 7x^2 - 28x \\
 & \qquad\qquad -11x + 44 \\
 & \qquad - \; -11x + 44 \\
\end{array}
$$

$f(x) = (x-4)(-2x^2 + 7x - 11)$ 1

d) when $x = 0$, $f(0) = f(0.2 \times 0) = 44$

$(0, 44)$

when $y = 0$

$(0.2x - 4)(-2 \times (0.2x)^2 + 7(0.2x) - 11) = 0$

$\Rightarrow 0.2x - 4 = 0$

$x = 20$

$(20, 0)$

1st term 2nd term

$f(0.2x) = (0.2x - 4)(-0.08x^2 + 1.4x - 11) = 0$

$x = 20$ is the only solution to $f(0.2x) = 0$ since 2nd term is < 0 when we put into $b^2 - 4ac \Rightarrow 1.4^2 - 4(-0.08)(-11) = -1.56$

f(0.2x) intersects at y-axis

f(0.2x) intersects x-axis

Point of intersection : $(0, 44)$ 1 and $(20, 0)$ 1

Question 16 continued

(Total for Question 16 is 11 marks)

TOTAL FOR PAPER IS 100 MARKS

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